Effect of Sublimation on Stagnation-Point Heat Transfer

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Gases flowing outward from the surface of a body reduce the energy transfer to that body. This mechanism is utilized in many cooling systems, especially for the cooling of aerodynamically heated surfaces. The outward flow of gas can be produced by injecting a gas or volatile liquid through a porous surface (transpiration cooling), by sublimation of the surface, or by more complicated processes as in the case of ablation. Ablation is the deterioration and removal of surface material by aerodynamic heating as in the case of the reentry of nose cones into the earth's atmosphere. During ablation the surface material partially decomposes into a gas, partially melts and is carried away by the air stream, and some of the surface material vaporizes. In all of these cooling systems only the vapor produced at the surface effects a significant reduction in the heat transfer coefficient to the surface. A surface which merely melts will not reduce the heat transfer coefficient to the liquid surface. The melting and decomposition processes, however, absorb energy at the surface and reduce heating of the interior of the body.

The purpose of the present experimental investigation was to determine the effect of the vapors produced at the surface upon the heat transfer to the surface. Specifically, the reduction in the stagnation-point heat transfer coefficient as a function of the mass of vapors produced was measured. For this purpose carbon dioxide models were placed in air streams with temperatures between 500° and 800°K. and velocities between 100 and 300 cm./sec. The profile of the model was photographed while it vaporized in the air stream. Subliming models of carbon dioxide were used in this study for three reasons:

1. All of the material removed from the model is vapor because there is no liquid phase for carbon dioxide at atmospheric pressure. Therefore, under steady state conditions the energy transferred to the surface equals the energy carried away by the vapors.
2. The properties of carbon dioxide are well known; therefore, the surface temperature and heat of vaporization are known.
3. The sublimation temperature is less than 200°K.; for this reason, high heat and mass transfer rates can be obtained without resorting to extremely high free-stream temperatures.

Brooke (*1*) and Weiss (*2*) used carbon dioxide models for heat transfer studies in supersonic air streams. These experiments produced no conclusive results regarding the reduction in heat transfer caused by the subliming carbon dioxide. Weiss did obtain good data with camphor and naphthalene.

Several theories predicting the effect of mass transfer on the stagnation-point heat transfer coefficient are presented in the literature; (*3, 4, 5, 6*) however, very little experimental information with which to verify these theories is available. Other authors (*7, 8, 9*) have developed theories concerning the effect of mass transfer although not specifically for the stagnation point. Mickley (*8*) also obtained experimental data for heat and mass transfer from a flat plate. An approximate theory presented by the author (*6*) relates the heat transfer coefficient and the mass transfer rate at the stagnation point by the following simple expression:

$$\frac{h}{h_o} = \exp\left(-\frac{1}{\pi N^2_{St}}\right)\left[1 + \text{erf}\frac{1}{\sqrt{\pi}\, N_{St}}\right]^{-1} \quad (1)$$

The values of all parameters except the stagnation-point heat transfer coefficient to a body without simultaneous mass transfer, h_o, which occurs in the Stanton number, can be computed from experimental measurements obtained with subliming materials. There are well-developed theories (*10, 11*) for computing h_o in laminar flow. The experimental air stream used in this investigation may have been turbulent; therefore, heat transfer coefficients without sublimation were measured with a copper calorimeter of the same size and shape.

DESCRIPTION OF APPARATUS

A sketch of the low-speed wind tunnel used in the present investigation is shown in Figure 1. Air from a mechanical compressor was filtered and reduced from 100 lb./sq. in. to slightly above atmospheric pressure by a pressure regulator. A surge tank downstream of the pressure regulator removed fluctuations in the air flow. An orifice meter built to standards of the American Society of Mechanical Engineers (*12*) was used to measure the mass flow of air. Water manometer number 1 in Figure 1, which has one leg open to the atmosphere, measured the air-stream pressure before the stream entered the orifice meter. Manometer number 2 measured the pressure drop across the orifice plate. These pressures were required to compute the mass flow of air.

The heaters and test section were contained in an insulated stainless steel tube which narrows from 6 to 3 in. in diam. through a conical section downstream of the heater section and approximately 1 ft. above the test section. The tube was oriented vertically with the hot-air stream moving downward so that gravity would not cause asymmetry in the flow when the tunnel was operated at low velocities. Three 16-mesh screens were placed in the flow between the test section and the heaters

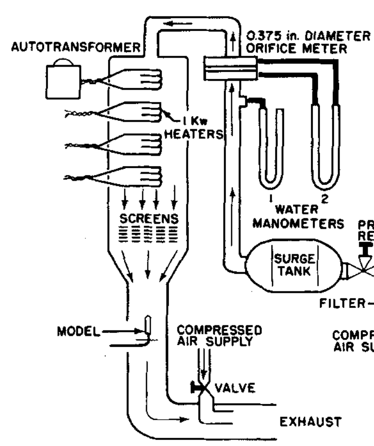

Fig. 1. Sublimation apparatus.

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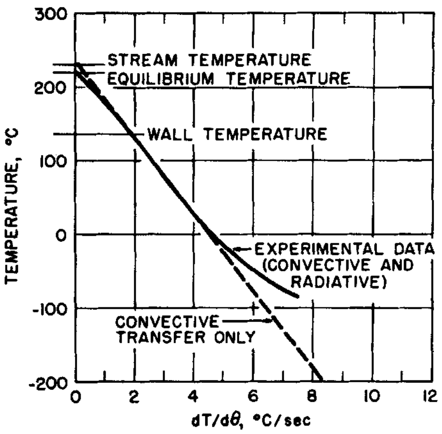

Fig. 2. Experimental calorimeter temperatures.

to prevent direct radiation from the heaters to the model and to damp out large-scale fluctuations in the velocity and temperature profile. The air stream was heated in the 16-in.-diam. section of the tube by four 1-kw. heaters. The power input to one heater could be varied by means of an autotransformer. With the proper setting of the pressure regulator and the heaters, velocities between 100 and 300 cm./sec. and temperatures between ambient and 800°K. were obtained in the test section.

A ¼-in.-diam. compressed air line terminating in the center of the exhaust duct formed a simple air-ejector system. When proper adjustment of the valve in the compressed air line was made, uniform flow past the opening in the test section was produced. The slight negative pressure maintained in the exhaust duct by the ejector eliminated the flow which normally would pass outward through the small opening in the test section. Flow in or out of this opening, if permitted, would have caused a large disturbance in the flow field at the test section. The test section was not airtight because models had to be inserted easily and a strain gauge attached to the model support required that support did not touch the tube. The strain gauge was designed to measure the mass of the model continuously; however, variations in air drag interferred with this measurement.

The air-stream temperature at the center line of the tube upstream of the test section was measured by a Chromel-Alumel thermocouple. The wall temperature was measured at the test section by a similar thermocouple attached to the outer surface of the tube to estimate the thermal radiation to the model.

The velocity at the center line in the test section was larger than the average velocity computed from the orifice meter measurement because of the growth of a boundary layer on the wall. The velocity at the center line was measured experimentally by a technique described by Roshko (*13*) which is based on the vortex-shedding frequency behind a circular cylinder normal to the stream. At a velocity of 400 cm./sec. the accuracy of this technique is comparable to that obtained with a Pitot tube and manometer, while at velocities below 400 cm./sec. this technique is more accurate. The probable error in the air-stream velocity is less than 2%. Since the heat transfer coefficient is proportional to the square root of velocity, the probable error in the heat transfer measurements owing to uncertainty in velocity is less than 1%.

HEAT TRANSFER WITHOUT SUBLIMATION

Hemispherical, 0.711-cm.-diam. copper calorimeters were used to determine the heat transfer coefficient to a nonsubliming body. The copper was mounted in the end of a glass tube with the same outside diameter as the hemisphere. This produced a smooth hemisphere cylinder which substantially reduced heat transfer to the rear of the calorimeter. A copper-constantan thermocouple attached to the flat surface of the hemisphere extended out of the glass tube. A copper-constantan reference junction maintained at 0°C. by an ice-water mixture was used. The output of the thermocouple circuit was recorded on a chart recorder. The polarity of the input to the recorder could be reversed by means of the double throw switch and thus enable negative voltages to be recorded. This was necessary because the copper models were cooled to the temperature of liquid nitrogen (approximately 76°K.) before being placed in the air stream. When the calorimeter was inserted in the hot air stream the first electromotive force recorded corresponded approximately to liquid nitrogen temperature. The electromotive force up to the time the recorder trace touched the electromotive force = 0 axis was negative. At that time the polarity was reversed. The remainder of the trace, which approached an equilibirium temperature asymptotically, was positive. The temperatures corresponding to the thermocouple electromotive force were obtained from a National Bureau of Standards table (*14*). In this manner heat transfer data in the absence of sublimation was obtained over the same temperature range experienced by the carbon dioxide models.

The data read from the recorder charts were plotted in the manner shown by the solid curve in Figure 2. This curve can be described by the energy balance

$$a \frac{dT}{d\theta} = T_s - T + b\,(T^4_w - T^4) \tag{2}$$

where

$$a = \frac{1.21\, mC}{Ah_o}$$

and

$$b = \frac{1.21\, F}{h_o}$$

The average heat transfer coefficient for a hemisphere was related to the local coefficient at the stagnation point using the theoretical data of Sibulkin found in a report by Korobkin (*15*). The temperature at $dT/d\theta = 0$ is by definition the steady state or equilibrium temperature. If radiation from the walls is an appreciable part of the energy transfer to a body, then the equilibrium temperature achieved by the body in the hot stream is between the air stream temperature, T_s, and the temperature of the surroundings, T_w. The solid curve shown in Figure 2, therefore, contains a radiation component of heat transfer. If the radiation term were negligible, that is, $b\,(T^4_w - T^4) = 0$, the equation of a straight line on a temperature vs. $dT/d\theta$ plot is obtained as shown by the dashed curve. The two curves will intersect at $T = T_w$ because there is no net radiation heat transfer when $T^4_w - T^4 = 0$. From these curves, a method for determining the average convective heat transfer coefficient in the presence of combined convective and radiative heat transfer is evident. With data of the type given by the solid line in Figure 2 and a knowledge of the free stream

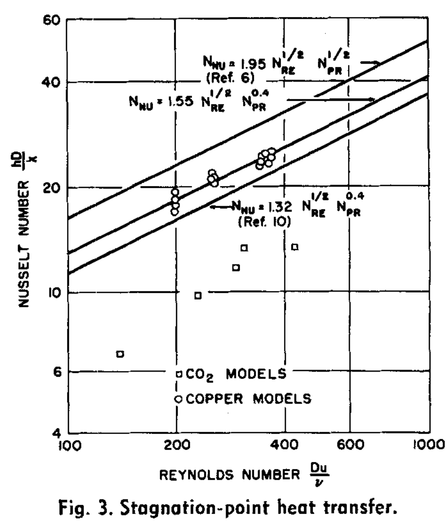

Fig. 3. Stagnation-point heat transfer.

and wall temperatures, the straight, dashed line can be drawn through the points $T = T_s$, $dT/d\theta = 0$; and $T = T_w$ on the solid curve. The stagnation-point heat transfer coefficient can then be computed from the slope of this line, which is 1.21 mC/Ah_o. The heat transfer coefficient for the data illustrated was

$$h_o = \frac{1.21\,mC}{Aa} = 0.00359 \text{ cal./sq. cm.-sec.-}^\circ\text{K.}$$

Dimensionless heat transfer rates without mass transfer predicted by potential flow theory (*6*) and the laminar boundary layer theory of Sibulkin (*10*) are shown in Figure 3, with experimental values obtained in this investigation.* The calorimeter data lie between these two theories, because slight turbulence which was undoubtedly present in the air stream employed in this experiment produced larger heat transfer rates than those predicted by the laminar boundary layer theory. The nonsubliming, heat transfer data can be fitted by the expression

$$N_{Nu} = 1.28\, N_{Re}^{1/2} \quad (3)$$

Since the variation of Prandtl number in the range of temperatures investigated is very small, an expression involving Prandtl number equivalent to those shown in Figure 3 could not be derived. However, by assuming a 0.4 power function of Prandtl number, the relation would be $N_{Nu} = 1.55\, N_{Re}^{1/2}\, N_{Pr}^{0.4}$.

HEAT AND MASS TRANSFER MEASUREMENTS WITH SUBLIMATION

The rates of sublimation of 0.762-cm.-diam. hemisphere-cylinder models of carbon dioxide were measured. The carbon dioxide models were made by filling a stainless steel mold with crushed dry ice and then compressing with a hand press. The mold consisted of three pieces which were cooled before each model was cast. The hemispherical nose and a small cylindrical hole in the bottom of the model for mounting were formed during the casting process. Models made in this manner were transparent and had specific gravities greater than 1.5. The specific gravity of pure carbon dioxide is 1.56. Upon removal from the mold, the models were either stored in a Dewar flask with crushed dry ice or mounted directly in the sublimation apparatus for a test.

The rate of heat transfer and sublimation at the stagnation point was determined by the rate of recession of the stagnation point on the carbon dioxide model. The distance to the stagnation point was measured on photographs taken at intervals during each test. A set of such photographs is shown in Figure 4. The horizontal bar in the photographs is exactly 2 cm. long and was used as a scale when making measurements. The bar also served as a fixed reference point from which the stagnation distances were measured. The photographs were made by withdrawing the model from the tunnel into the view of a Polaroid camera, taking a picture, and then returning the model to the hot air stream. This operation required from 2 to 3 sec. The time at which the model was withdrawn and returned to the test section was recorded electrically on a chart recorder by a strain gauge attached to the mechanism on which the model was mounted. The time the model spent out of the air stream being photographed was subtracted when computing the total sublimation time. Several tests were run at identical air stream conditions; but the time spent out of the air stream was varied by taking different numbers of photographs. In this manner it was ascertained that interrupting the sublimation process did not introduce appreciable error in the data.

* Tabular material has been deposited as document 7582 with the American Documentation Institute, Photoduplication Service, Library of Congress, Washington 25, D. C., and may be obtained for $1.25 for photoprints or for 35-mm. microfilm.

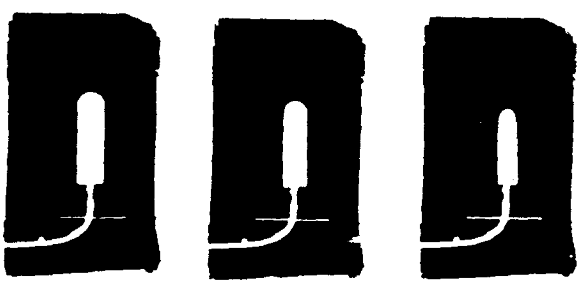

Fig. 4. Carbon dioxide model during experiment.

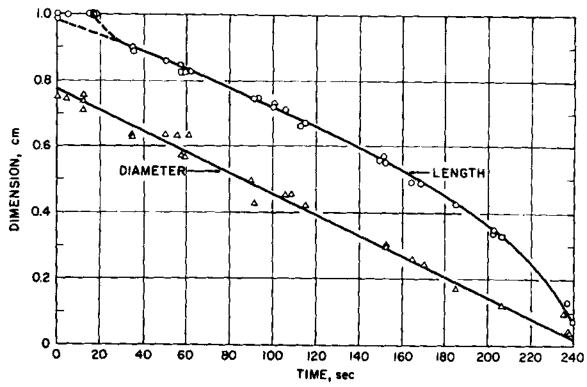

Fig. 5. Model length and nose diameter vs. time.

The nose diameter and the length of the model obtained from the photographs by means of a microfilm viewer were plotted as a function of time. The model lengths which indicated the recession of the stagnation point were reduced by a constant to give an initial length of 1 cm. Figure 5 illustrates the results of five tests, which were obtained in a stream with an average velocity and temperature of 136.04 cm./sec. and 492.7°K. The nose diameter was difficult to measure because the model did not remain a hemisphere cylinder, but assumed a shape with a radius of curvature which increased gradually with distance from the stagnation point. This accounts for the scatter of points about the diameter curve in Figure 5.

The convective energy transfer, $h\,(T_s - T)$, and radiative energy transfer, q_r, to the solid-vapor interface is equal to that absorbed by sublimation, GL_s, and that conducted into the solid, q_c. The mass transfer rate at the stagnation point, G, can be obtained from the observed rate of recession of the carbon dioxide surface. A combination of the appropriate energy and conservation equations gives

$$-\frac{dx}{d\theta} = \frac{h(T_s - T) + q_r - q_c}{L_s\, \rho_{CO_2}} \quad (4)$$

The terms on the right-hand side of Equation (4), with the exception of h, are constant during most of a single test. If it is assumed that q_r and q_c are small and h/h_o is constant, it is found from Equation (3) that the recession rate is inversely proportional to the square root of the nose diameter. Theoretically there is a slight variation in h/h_o with change in nose diameter; however, this variation is less than the uncertainty in the data.

The experimentally measured nose diameters were fitted with a straight line. When the expressions for nose diameter vs. time and recession rate vs. nose diameter were combined, the equation for model length as a func-

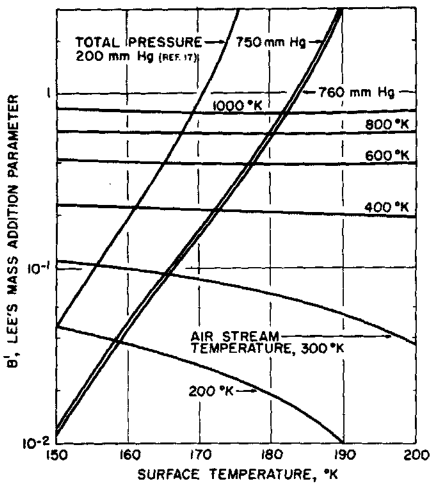

Fig. 6. Carbon dioxide sublimation temperature vs. free-stream conditions.

tion of time was established. This equation was used to fit the model length data. The curves fitted to the experimental data from five tests by the technique described are shown in Figure 5. The good fit to the length data indicates that the assumptions employed are reasonable. The constants obtained by fitting the data of Figure 5 as well as the reduced data are available in the literature (*16*). The values of model length tended to scatter in the neighborhood of $\theta = 0$ for all tests. As will be shown later, the equilibrium surface temperature of a carbon dioxide model in the hot air stream was less than that of carbon dioxide maintained under 1 atm. of its vapor. For this reason the model temperatures fell when the models were removed from the Dewar flask in which they were stored and inserted in the hot air stream. This thermal energy was absorbed by the vaporization process and caused an abnormally high sublimation rate during the first few seconds as shown in Figure 5. During this initial period the assumption that $q_c = 0$ did not apply. After approximately 20 sec., the model reached equilibrium with the air stream, and q_c was constant and small compared to $h(T_s - T)$.

The surface temperature of the model was not measured experimentally, but was computed by using Lees' (*7*) technique. If it is assumed that the momentum, thermal, and mass transfer boundary layers are similar, the carbon dioxide concentration at the surface and, consequently, the surface temperature can be computed without a knowledge of the absolute transfer rates. Kubota (*17*) used this method to compute surface temperatures of subliming models of water, carbon dioxide, and camphor. This computation has been repeated for the present range of interest and is shown in Figure 6. The models in the Dewar flask were maintained at 194.7°K., which is the sublimation temperature of carbon dioxide under 1 atm. of its vapor. Figure 6 indicates the surface temperature may be lowered as much as 30°K. when the model is introduced into the air stream. An appreciable error in this effect will, however, produce only a small error in the temperature difference, which varied from 317 to 700°K. The heats of sublimation of carbon dioxide at each surface temperature were obtained from Brooke (*1*).

When Equations (3) and (4) are combined with the definitions of the dimensionless parameters, one obtains the equation used to reduce the experimental data:

$$\frac{h}{h_o} = \frac{D(GL_s - q_r)}{1.28\, kN_{Re}^{1/2}(T_s - T)} \qquad (5)$$

The heat transfer ratios, h/h_o, computed by means of Equation (5) are plotted vs. the mass transfer parameter, $\frac{G N_{Re}^{1/2}}{\rho u}$ in Figure 7. The five data points were obtained using twenty-three carbon dioxide models. The large uncertainty in some values of h/h_o resulted from uncertainties in q_r. The model received radiant energy from the inner walls of the test section which varied in temperature from that measured close to the model to very nearly the stream temperature. Only the wall temperature of the test section was measured. Radiant energy from the screens which were at the stream temperature also was incident on the model. The energy flux from this complicated geometry and temperature distribution was not computed exactly because of insufficient information. The limits of uncertainty shown by the bars in Figure 7 were computed by assuming that the radiation, q_r, was received from surfaces at two extreme temperatures, the free stream temperature and the wall temperature nearest the model. The effective temperature of the radiating surfaces was closer to the test-section wall temperature than the free-stream temperature; therefore, the actual values of h/h_o lie near the lower end of the bars in Figure 7. Probable values of h/h_o based on these extreme values of q_r are shown by the curve.

DISCUSSION OF RESULTS

For comparison, the results obtained by Weiss (*2*) are plotted in Figure 7. The agreement is good in light of the large differences in free-stream conditions. Weiss' data were obtained at Mach 3.5 and Reynolds numbers of 4×10^5 to 10^6 with naphthalene and camphor. The agreement is considered good because h/h_o is very sensitive to the absolute magnitude of two independently measured heat transfer coefficients. This is evident from the fact that Brooke (*1*) obtained values of h/h_o greater than 2 and therefore did not fall on Figure 7. It is not clear to the authors how values of h/h_o greater than 1 are possible. A problem encountered in previous experiments was that the models became rough during the test. This will give values of h/h_o greater than 1 if h is based on the area of a smooth model. In the present experiment the

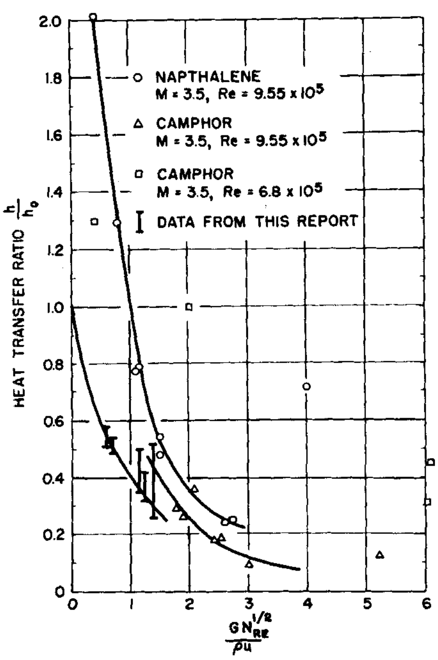

Fig. 7. Comparison of experimental sublimation data.

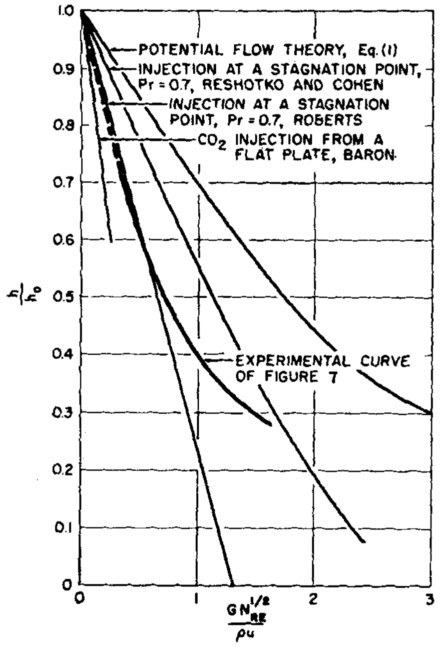

Fig. 8. Comparison of experiment with theory.

models remained smooth and axisymmetric until late in the test. As shown in Figure 4, the models were particularly smooth in the stagnation region. This may be a result of two factors. The models are not subject to erosion at the relatively low velocities employed, and the small models used in this investigation could be formed under very high pressures giving a strong, homogeneous model.

The results are compared with the theories available in the literature in Figure 8. Potential flow theory exhibits the same shape as the experimental curve; however, this theory predicts mass transfer rates which are twice the experimental values. The theoretical calculation of Roberts (3) falls close to the experimental data for values of $h/h_o > 0.5$. Roberts obtained a linear relationship between the heat transfer ratio and mass transfer because of the assumptions used concerning flow in the boundary layer. The theory is either discontinuous at $h/h_o = 0$ or it predicts negative heat transfer rates. This is possible if the enthalpy of the fluid injected is included in the convective heat transfer. In the present experiment there can be no sublimation without convective heat transfer to the interface. Therefore, the experimental curve in Figure 8 will approach $h/h_o = 0$ asymptotically.

At high mass transfer rates many of the usual boundary-layer approximations do not apply. This is the region in which Roberts' theory and the expeiimental data diverge. These approximations are that pressure gradients and certain derivatives of velocity normal to the surface are negligible. More data should be obtained in the region $h/h_o < 0.3$. In the present tests radiative heat transfer became comparable to the convective heat transfer at low values of h/h_o; therefore, the apparatus could not be used to investigate this region.

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NOTATION

A = area
C = specific heat of model
C_p = specific heat of air at constant pressure
D = diameter
F = radiation parameter for apparatus
G = mass transfer rate per unit area
h = stagnation-point heat transfer coefficient
h_o = stagnation-point heat transfer coefficient without mass transfer
k = thermal conductivity
L_s = latent heat of sublimation
m = mass of model

$$N_{Nu} = \text{Nusselt number} = \frac{hD}{k}$$

$$N_{Pr} = \text{Prandtl number} = C_p\mu/k$$

$$N_{Re} = \text{Reynolds number} = \frac{Du}{\nu}$$

$$N_{St} = \text{Stanton number} = \frac{h_o}{G\,C_p}$$

q_c = rate of heat conduction
q_r = rate of radiant heat transfer
T = model temperature
T_s = air-stream temperature
T_w = wall temperature
u = free-stream velocity
x = length of model
θ = time
ν = kinematic viscosity
ρ = density of free stream

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